

# **Review of the Everyday Mathematics Curriculum and Its Missing Topics and Skills**

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## **Introduction**

This paper will review the main teaching philosophy of the Everyday Mathematics, the math curriculum under consideration by the Oak Ridge School Board of Education for adoption for k-5 grade levels system wide.

In spite of some attractive approaches undertaken by the Everyday Mathematics (EM), the topics and skills it leaves out will be seriously detrimental to the students in rendering them not being fully prepared to handle the learning of higher mathematical topics in middle, high and finally the college years. These deficiencies alone warrant the unsatisfactoriness of the Everyday Math curriculum for the Oak Ridge schools (for any schools for that matter). However, in addition to these deficiencies, EM stresses and adopts the attitude of shunning the standardized algorithms that have been developed through out the ages for the basic operations of addition, subtraction, multiplication, and division. Not only are they not taught per se in the curriculum, but EM wants the students to choose her own method of preference among several alternative algorithms (usually cumbersome) including those the student has come up with, under all circumstances to solve an operation procedure. This emphasis of free choice is pervasive throughout the curriculum (k-6) and is the hallmark of Everyday Mathematics.

## **Flaws of Two Major Longitudinal Studies Claiming the Effectiveness of Everyday Mathematics**

Before I present the topics and materials not covered by the Everyday Mathematics curriculum, I would like to present briefly the serious flaws associated with two major longitudinal studies, performed by an education group at the University (Achievement Results for Second and Third Graders Using the Standards-Based Curriculum Everyday Mathematics, by K. Fuson, W. Carroll, and J. Drueck, Journal for Research in Mathematics Education, Vol, 31 (3), May, 2000, 278-295; Achievement Results for Fourth

Graders Using the Standards-based Curriculum Everyday Mathematics, by W. Carroll, and K. Fuson). More detailed analysis of the flaws of these two studies are presented as the last section of this report. **Professor John Ray, a full professor in Evaluation, Assessment, and Testing Development at the Department of Education, The University of Tennessee, has confirmed my analysis of the flaws associated with these studies.**

The fatal flaws of the study are summarized below:

1. The test used to compare the performance of kids taught under the Everyday Mathematics and others (termed U. S. traditional students) is one made up by the evaluators themselves, with problems pulled partially and selectively from several existing tests identity of some of which are unknown. A total of four tests were given in the two studies reported, but the identity of only two of the tests is given (the NAEP test) from which selected problems from subtests are pulled. The categories of the test areas are unclear as to its completeness in representing the overall mathematical competency.
2. The test group is consisted of Everyday Math kids of the size of 343 to 238 of 2<sup>nd</sup> and 3<sup>rd</sup> graders, and 170 of 4<sup>th</sup> graders, respectively of completely known background; while the control group, in one case is composed of just 29 students from just one San Francisco school (may even be from just 1 class) and in the second, composed of 1800 students from a pool of 18,000+ students of unknown origin, while in the third study, the control group used is of 2000+ each of students from a 1990 and 1994 NAEP test, in which the background of both control groups are completely unreported and thus unknown. Who are they, and are they in every way similar to the test group except for the method of instruction of mathematics?
3. How can meaningful conclusions be extracted from results using control groups of very unequal mix of background to the test group and of size of only 29 from one school, and claiming these 29 represent the mix of the 'U.S. tradition kids'; while in the other group, the attributes of the 1800 (or 2000+) students are completely unknown. Results from these two studies are completely useless to support the superiority of the Everyday Mathematics curriculum.

Excerpts and examples will be taken directly from the Everyday Math curriculum material to illustrate and support this review of the curriculum. When available, websites containing the complete source referenced in this article will be given for the readers to follow up to learn more about the curriculum and the controversy it engendered. I invite readers to log onto the website, <http://www.lit.net/orschools/> (prepared by my spouse and me) which contains links to other relevant sites, with a short phrase explaining the relevance of each link and the nature of the materials contained within. (This last part is still under construction.)

## **History of the Emergence of the Constructivist Mathematics Curriculum**

In an attempt to improve the general mathematics instruction in the U.S., the National Council of Teachers of Mathematics (NCTM, the highest and most influential professional organization representing the teachers of mathematics in the US) revised its guidelines and evaluation standards for mathematics teaching in 1989. The new guidelines are what brought on the proliferation of the various Constructivist curriculum. The new NCTM standards are regarded by professional mathematicians as watered down and among other things, inadequate in its not teaching the standard mathematical algorithms of addition, subtraction, multiplication, and division. (<http://www.wgquirk.com/NCEE.html> ). Subsequently, the National Science Foundation (NSF) funded the creation and evaluation of several math-teaching curriculums in accordance to the new NCTM standards, and in 1998, NSF formally endorsed 10 curriculums as either 'exemplary' or 'promising'. (<http://mathematicallycorrect.com/nation.htm#doesham> ) Everyday Math is among these. Since then, commercial publishers have emerged publishing and promoting these various curriculums. In April of 2000 (<http://mathematicallycorrect.com/nation.htm#doesham> ) a group of 5 professors of mathematics at several leading California and other universities, along with 200 cosigners of professors of mathematics, science, engineering, and some leading school administrators and education specialists, sent an open letter to Secretary Richard Riley, then secretary of US Department of Education, asking the withdrawal of the endorsement. Secretary Riley declined and the controversy intensified. The list of cosigners include four Nobel laureates and 3 Field Medallist (the highest award in mathematics research).

## **Revolt to the Adoption of the Constructivist and Everyday Math around the Country**

During the last several years, various parental groups around the country were formed, with the specific mission of fighting against their respective school district board of education to stop the use of the various constructivist math curriculum in their schools. (too numerous to mention; check in <http://mathematicallycorrect.com> under the heading of California and The Nation and [http://www.educationnews.org/nyc\\_math\\_warsnyc\\_honest\\_open\\_log.htm](http://www.educationnews.org/nyc_math_warsnyc_honest_open_log.htm); <http://mathematicallycorrect.com/citylist.htm>) The most active effort is in the state of California, who has widely adopted the constructivist math teaching throughout the k-12 grade levels at numerous large school districts, in particular, at Los Angeles, San Diego and Palo Alto. The state of California has seen a drastic decline of the math achievement scores in computation declining from the 80% of national normed ranking to high 40% over the years (<http://www.rahul.net/dehnbases/hold/release-trd.html> ). In 1998, the State of California, under the guidance of professional mathematicians and scientists, revamped the California State Mathematics Standards and developed a list of math textbooks recommended and not recommended for adoption. (<http://mathematicallycorrect.com/adopt.htm> ). Explicitly among the 'not recommended' list is the Everyday Mathematics. It is in the 'not recommended' list, not just because it does not have a student

textbook, but mainly because of its inadequate content, as well as its teaching philosophy. (<http://mathematicallycorrect.com/programs.htm>; <http://www.mathematicallycorrect.com/books.htm>) In addition, the Texas Department of Education has also undertaken a review of the mathematics curriculum and developed a list of recommended and not recommended textbooks. Explicitly among the 'not also the Everyday Mathematics. (<http://www.tppf.org/pau/1999/pau052499.html>) In particular, in the district of Plano, Texas, the parent group sued their school board for failure to provide alternate math teaching to the Everyday Math. (<http://mathematicallycorrect.com>). Furthermore, in Reading Massachusetts, a group of concerned parents fought hard to have Everyday Math discontinued (<http://members.aol.com/rmandell/CPR>) to no avail. The result is a migration of a lot of kids to private schools and private tutoring (private communication). Other numerous report of mass unrest to the constructivist math abound and are reported at the website of [www.mathematicallycorrect.com](http://www.mathematicallycorrect.com), cofounded by several scientist to provide a clearinghouse of information about the reaction and experiences from around the country in regard to the adoption of constructivist math curriculum at various school districts.

### **What Are the Teaching Philosophy and Practices of the Everyday Math Curriculum**

At the mathematicallycorrect.com websites, review of various constructivist math curriculums, including those for the elementary, middle and high schools are available. In this article, I will review the Everyday Math curriculum in specific and detail what some of the important topics and skills deliberately left out are.

The teaching philosophy of Everyday Math (EM) is that the kids, through active participation and experimentation, 'construct' the algorithms required to solve a mathematical operation, such as the addition or multiplication of 2 multi-digit numbers. In addition, the main thrust of the teaching of EM is in the over emphasis of the development of the concepts of mathematics and the associated operations, such as asking the question, 'what is the meaning of 15 divided by 5', or 'what do we mean by 3 divid emphasis almost to the exclusion of requiring also of the mastery of the skills required to actually perform, with efficiency, the operations at hand. The few alternative algorithms favored by the curriculum are usually fairly cumbersome, and are absolutely not practical to be extended to handle larger problems, when the numbers involved are big or nasty, such as 5-digit numbers for addition and multiplication, or 13/17 as the divider in a problem of division by fraction. In addition, an over reliance on calculator to handle the actual computations whenever the calculations become beyond 'mental calculation' level is evident in the curriculum. I will quote passages directly from the EM published booklet to teachers to support my claim.

The EM folks have an aversion to teachings of standardized algorithms to students, algorithms that have been arrived by centuries of teachings and practicing of mathematics. They pride in having the students develop their own, so they would feel a sense of 'ownership'. This approach would be fine at the

beginning of the introduction of a new concept. But toward the end, the standard algorithm as we know them should be taught and emphasized that they should be the ones used in the future to solve problems, because it is usually the most efficient and the most reliable algorithm, guaranteed to work under all situations, and can be extended directly with equal ease to bigger problems. Although they are not adverse to teachers showing the slow or recalcitrant learner the standardized algorithms they are under no obligations to show any. The Everyday Math people support arguments made by the constructivist reformers (<http://www.edweek.org/ew/1994/20lein.h13>) to abandon computational algorithms by paper-and-pencil, as well as 'to have children sharing their own invented algorithms rather than for teachers continuing to teach the standard paper-and-pencil algorithms as such.(Everyday Mathematics Teacher's Reference Manual, Grades k-3, Everyday Learning Corporation, Chicago, Ill. 1999, ISBN 1-57039-515-2).

'The following is a quote from the Teachers Reference Manual, k-3, pages 91

*Everyday Mathematics believes children should be encouraged to invent and share their own ways of doing operations, rather than having everyone learn the same standard algorithms at the same time. Learning algorithms too early may actually inhibit the development of children's mathematical processes and cause them to miss out on the rich experiences that come from developing their own computational strategies. ...'*

*'Everyday Mathematics hopes that your children will arrive at their own preferred ways of performing computations. ... There is often outside pressure for children to master specific computational algorithms. Indeed, this may be your own personal preference. ... The program's aim is to help teachers, not to impose ideas or demands on them.'*

The EM's emphasis of requiring the students to develop the understanding of a mathematical concept, such as multiplication, is an excellent one, as should be the case no matter what the teaching method is. Therefore, a considerable amount of time and effort in EM is spent in showing the students what are meant by the various mathematics topics introduced. After showing various EM preferred algorithms for carrying out the required computations as examples, the students are asked to invent their own also and to start using whatever they feel comfortable in all future problem solvings. However, EM stops short of introducing the standard paper-and pencil algorithms and emphasizing that these algorithms are reliable and guaranteed to work under all circumstances, and are the most efficient paper-and-pencil algorithms when the problems and numbers become large or nasty (decimal, rational, irrational or complex numbers)

Reviews of the shortcomings of the Everyday Mathematics curriculum by various professors of mathematics at leading universities are available at the following web addresses:

<http://www.mathematicallycorrect.com/everyday.htm>

<http://mathematicallycorrect.com/programs.htm>

<http://www.mathematicallycorrect.com/books.htm>

## **The Preferred Algorithms for doing the Four Standard Mathematical Operations**

The preferred algorithms by the EM people for each of the four standard math operations, addition, subtraction, multiplication, and division, will now be briefly presented. For more complete examples, please see the Appendix which are pages directly from the Everyday Math Teachers Manual for k-3, and 4-6, in explaining their teaching approaches.

### **Addition**

- **Left-to-right algorithm:** addition starts from left to right digit by digit. Then each digit of the sum is adjusted to take care of the carry-overs. It takes multi rows of intermediate results to add two numbers.
- **Partial-sums algorithm:** add the digits in each column of digits with their place values, then add up the partial sums. To add 2 5-digit numbers, addition of 5 partial sums is required.
- **Rename-Addends algorithm:** In this algorithm, the numbers to be added are to be changed to nicer numbers ending in 0's to result in simpler operations. For example, in adding 267 to 481, 267 is first changed to 270 by adding 3 to it. This is followed by subtracting 3 from 481 to change it to 478. This step is now followed by changing the 270 to 300, followed by changing 478 to 448. Finally the new numbers of 300 and 448 are added to give 748. The whole idea is to change 270 to the nearest hundreds so as to make the final addition easier, involving 0's. What happens if the first number is 217 instead of 267. This algorithm would call for changing 217 first to 220, then to 300, thus involving subtracting first 3 then 80 from the second number before the final addition. Isn't subtracting 80 from 478 almost as hard as the original addition problem? Why go through 5 addition and subtraction steps to add just 2 3-digit numbers. Imagine how many steps one has to go through to add say 2 5-digit numbers (at least 9 steps). Is this an efficient procedure for generalizing to bigger problems? What about addition of decimal numbers?

### **Subtraction**

- **Add-Up algorithm:** Starting from the subtrahend (the number to be subtracted), add up to the nearest tens, then the nearest hundreds, and thousands, until you reach your number that you are subtracting from. For example to do 932-356, you change 356 to 360, 400, 900, then to 932, by recognizing you need to add 4, 40, 500, and 32 respectively to arrive at 932. So the answer is arrived at by adding up these four numbers to get 576. This procedure involves 5 intervening steps.

- Left-to-right algorithm: It also involves several intervening steps. For example, the problem of  $(932-356)$  involves  $932-300$ , followed by  $632-50$ , followed by  $582-6$  to get 576. But isn't the step involving  $632-50$  almost as hard as the original subtraction step? If you follow strictly the preferred algorithm of Everyday Math, to do  $632-50$  would involve another multi-step procedure to get the answer!
- Rename-subtrahend algorithm: For example, to do  $932-356$ , one changes 356 to 360, then to 400, and at the same time changing 932 to 936 then to 976, followed by doing  $976-400$  to get the answer of 376. **In order to use this algorithm, the student has to 'memorize by rote' that when doing subtraction, in order to preserve equality of the answer, one has to add the same number to both the subtrahend and the to-be-subtracted number. Whereas, in doing addition, the similar algorithm involves 'remembering by rote' adding and subtracting the same number to the addend and to-be-added number in order to preserve equality. Isn't this required memorization an added burden to the students?**
- **Two Unusual subtraction algorithms: Please see the appendix for detail. The author is too tired to explain it here.**

## Multiplication

- The partial product algorithm: Multiplication of two numbers is broken into the sum of products of intermediate multiplication steps. For example,  $56 \times 23$  is broken into the sum of  $50 \times 20$ ,  $50 \times 3$ ,  $6 \times 20$ , and  $6 \times 3$ . Imagine multiplying two 4-digit numbers. A total of 16 partial products need to be summed to arrive at the final answer, in addition to writing out all the intermediate numbers that make up the original numbers.
- The lattice method: Partial digit by digit products are entered into cells of a rectangular array, followed by addition of laterally aligned numbers. Multiplication of 2 3-digit numbers would involve entering 9 numbers into the appointed cells of an array followed by several rounds of lateral addition of several numbers.
- The old Egyptian algorithm: One of the two numbers to be multiplied is first written as the binary representation of the sum of numbers involving only powers of 2. (e.g.  $25 = 1+8+16$ ), while the other number is to be doubled at several successive steps, as many steps as the highest power of 2 involved in the binary sum of the first number. Finally, the overall product is the sum of the intermediate doubles of the second number. Besides being very very cumbersome the big question here is, how would one go about figuring out the binary representation of one of the numbers to be multiplied, a necessary prerequisite before the process can begin. This is a complicated task in itself, a more involved task than the original task of multiplying 2 numbers. And we expect elementary school kids to know it? To add insult to injury, in the 5<sup>th</sup> grade Everyday Math curriculum, the Egyptian algorithm

is not even explained to the students, but rather two examples involving 2 2-digit numbers are presented and the students are asked to figure out what's going on. I have to report that my husband and I, both have advanced degrees in physics and engineering, couldn't figure out how it is done right away. We finally cheated and looked in the teacher's manual. Even after that, we still had to spend some time figuring out why some of the intermediate numbers are crossed out, and what the associated rules for crossing out are. Everyday Math definitely prefers the partial product algorithm.. In homework problems, students have problems assigned using all these approaches. At no place is the standard algorithm that you and I know presented. At the fifth grade level, only 2-digit multiplication is presented and required. One would think that by the 5<sup>th</sup> grade, multi-digit multiplication should have been taught and its mastery required.

## **Division**

- Partial quotient algorithm: The standard long-division paper and pencil algorithm is not taught completely. In what little is taught in division by Everyday math, involves the following: The number to be divided is first broken into the sum of smaller numbers, each is an easily recognized multiple of the divisor. Then the sum of the partial quotients is to be added to get the final answer. In the fifth grade Everyday Math, division by 1 digit only is introduced and taught. The problem with this partial quotient algorithm is that it can't be generalized to multi-digit divisors. For instance, in  $973/27$ , according to this method, one has to recognize what partial number contained in 973 is a whole multiple of 27. I would be hard pressed to recognize that right away without first doing some paper-and-pencil calculation using method traditionally taught to me. (See the appendix Figures 15, 16 and 17 in which the EM folks claim that learning long-division algorithm is a waste of time.)

All the alternative algorithms the Everyday Mathematics curriculum suggest the teachers to teach (as shown above) are usually much more cumbersome than the standard algorithms you and I are familiar with, and they are almost all impractical to be extended to problems involving bigger numbers. They certainly would be infeasible to be employed in multiplying and division problems involving decimal numbers.

## **Other Important Traditional Skills not Taught in the Everyday Math Curriculum**

- Long Division: The mechanics of long division is not taught at all. The claim is that calculator would do the job nicely. The following is a paragraph taken verbatim from the Everyday Math Teacher's Reference manual for 4-6 (page 136).

*The authors of Everyday Mathematics do not believe it is worth the time and effort to fully develop paper-and-pencil long division algorithms for all possible whole number, fraction, and*

*decimal problems. Mastery of the intricacies of these algorithms is a huge undertaking, and one that experience tells us is doomed to failure for many students. It simply does not seem wise..., while nearly anyone can quickly and accurately find a quotient with a calculator.*

The problem here is that in learning the long-division algorithm, one is not just interested in getting a numerical answer, but rather in learning the actual algorithm in carrying out the division, because a host of important concepts and skills later on depend on having learned the longhand division skill. Professor Wu and Professor Klein both wrote very nice papers explaining the role of long division in k-12 as well as college curriculum, and the consequences of not teaching it.

(<ftp://math.stanford.edu/pub/papers/milgram/long-division/longdivisiondone.htm>,

<http://mathematicallycorrect.com> link to Wu's papers). To mention just few of the topics involving understanding the long hand division algorithm, they include rational numbers and the origin of decimals with repeats (e.g.  $1/3 = 0.3333\dots$ ), infinite series, convergence, polynomial long division, polynomial factorization, partial fraction expansion, approximation of functions, Taylor's Series expansion and approximations, the use of Laplace transforms in solving linear differential equations. These mathematical concepts and skills are involved in every high school mathematics course starting from pre-calculus and in every freshmen and sophomore algebra and calculus class in college.

- Division by Fraction: The standard algorithm of invert-and-multiply are shunned and not taught. As a matter of fact, from reading the abundant literature on the web ([www.mathematicallycorrect.com](http://www.mathematicallycorrect.com)) critical of the shortcomings of the constructivist math teaching approach, the traditional invert-and-multiply algorithm of fraction division has been cited by the constructivist reformers as the epitome of the 'drill-and-kill' traditional mathematical teaching. In the fifth grade Everyday Math, division by fraction involves only fraction of  $1/n$  in the denominator (where  $n$  is a low whole number, e.g.  $1/4$ ). The idea is that the students are to understand the concept of division by fraction as to mean 'how many  $1/4$  pieces are there in 3 (as in  $3/(1/4)$ )', and in using manipulatives by visual inspection, one is to arrive at the answer of 12 (there are four  $1/4$  pieces in a whole piece; therefore, there are 12  $1/4$  pieces in 3). What happens when the problem is  $19/(3/17)$ ? In the Teachers' Reference Manual Grades 4-6 (page 137), it explains the invert and multiply rule for division by fraction, only as an explanation that the teachers can use when asked by students who would be curious about how divisions by fractions are handled; or perhaps because their parents may have taught them on the side and they would ask their teachers about it, seeing the school is not teaching the kids this important skill.
- Multiplication of two decimal numbers is hardly mentioned in the 5<sup>th</sup> grade curriculum. In the sixth grade Everyday Mathematics, the students 'are encouraged to find their own algorithms for multiplying decimals in the same way they have been asked to find algorithms for operations with whole numbers and fractions'. (Everyday Mathematics Teacher's Reference Manual, Grades 4-6 page 135)

## **Ignoring a Large-scale Education Experiment Which Showed Direct Teaching Is More Effective**

In the history of education research in the U.S., the largest one ever conducted is probably the Project Follow Through experiment, a federally funded longitudinal study of more than twenty different approaches to teaching economically disadvantaged k-3 students. The experiment lasted approximately 9 years from 1967-76. Project Follow Through involved 70,000 students in more than 180 schools. (<http://mathematicallycorrect.com/honestft.htm>; <http://darkwing.uoregon.edu/~adiiep/ft/151toc.htm>; <http://www.edexcellence.net/library/carnine.html>) The teaching approaches being evaluated fell into two main categories: those based on child-directed construction of meaning and knowledge, and those based on direct teaching of academic and cognitive skills. The child-directed is similar to the present day constructivist approach, while the direct teaching is comparable to the present day labelled traditional approach. The Data gathered were analyzed by 2 independent groups—Stanford Research Institute and Abt Associates. Students were tested using the standardized Metropolitan Achievement Test in four categories, total reading, math, spelling and language. The bottom line is that only the Direct Instruction teaching approach, the participating students scored near or at or close to the national norms in all four categories. The four major teaching models in the child-directed education, in which the constructivist/discovery learning model was one, often performed worse than the control group, in spite of tens of thousands of additional dollars given to each class room each year, involved in the experiment. Further research on the Directed Instruction model showed that it worked in a variety of communities for a variety of groups with differing social economic levels. In 1999, almost 2 decades late, some of the nations' leading education organization confirmed the effectiveness of the Direct Instruction approach. However, to a large extent findings from Project Follow Through are ignored by the teaching community. The Carnine paper reports (the previous reference) that before Project Follow Through the constructivist approach was extremely popular, and they remained obstinate to change despite the finding of the Project Follow Through. Effort by school districts to return to the Direct Instruction format was often discouraged by education organizations. Of course, during the past ten years, a renewed resurgence of the proponents of the constructivist approach has taken place. It is such a shame that given the vast amount of evidence from the massive Project Follow Through experiment, the education community wants to ignore it and hold onto a believe which has been shown to be largely ineffective.

## **Two Longitudinal Studies Claiming Improved Math Achievement Performance by the Everyday Math Students are Seriously Flawed**

To school officials, and to the majority of the public, the bottom line of any math teaching is in the students scores in math achievement test, hopefully, a test of nationally recognized stature including all aspects of competency, especially in computational skills. Solid reports on test scores comparing scores before and

after adopting Everyday Math, are far and few in between, although some reports reporting either improvement or decrement in math achievement test scores are available, but quite a few of them are with serious shortcomings in the assessment of the design methodology. Test score improvement cited by the literature put out by the Everyday Learning Corporation (publisher of the Everyday Mathematics) notwithstanding, at least two major whole article reports on favorable longitudinal study outcomes of the Everyday Mathematics curriculum are seriously **flawed** in their test design, thus rendering the conclusion useless. Let me just briefly explain the type of flaws exhibited by two of the longitudinal studies (*Achievement Results for Second and Third Graders Using the Standards-Based Curriculum Everyday Mathematics*, by K. Fuson, W. Carroll, and J. Drucek, *Journal for Research in Mathematics Education*, Vol, 31 (3), May, 2000, 278-295; *Achievement Results for Fourth Graders Using the Standards-based Curriculum Everyday Mathematics*, by W. Carroll, and K. Fuson). In these two reports, conducted with similar assessment approaches, led by a Northwestern University group, funded by NSF, results are presented on the comparison of 100+ each of 2<sup>nd</sup> and 3<sup>rd</sup> graders (in the first report) and 170 4<sup>th</sup> graders (in the second report) in 11 and 8 Chicago area schools respectively who have been taught under the Everyday Math (EM) curriculum for various number of prior years with various other US 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> graders on a math achievement test, prepared by the Northwestern University group itself by selectively choosing problems from existing tests.

In general, it is universally agreed that the requirement for doing good research in assessing the effect of two different treatment methods, the two populations tested have to be kept, as much as possible, similar in every other aspect, except for the treatment method under comparison, such that any difference observed between the two groups can be attributed to the different treatment method being assessed. Thus in comparing the effect of teaching using the Everyday Math curriculum versus a traditional curriculum, the two groups of students being compared must be similar in all respect except for the method of math instruction. Given this as the requirement, the Fuson study is flawed in that the characteristics of the two groups being compared are unknown (maybe the authors know the characteristics of the test group but choose not to elaborate them; but the control groups' attributes are certainly not obtainable because the authors don't know exactly who they are; if they did, they certainly chose not to report their profiles to ascertain their being comparable to the test group in all other attributes.) Therefore, the conclusion drawn by the authors that EM kids performed better than US traditional kids are to be regarded as useless, at best. Let me relate the details of the two studies now. The first study of the first report involves 392 second graders, but test performance from only 343 of these students are analyzed, because these kids are also the ones involved in the study of first graders done in the previous year. These students are from 22 classes in 11 schools in some school district (One can't tell from the paper, although it is probably the Chicago area) including 'urban, suburban, and rural or small town schools, and the student populations ranged from low-income to affluent' according to the authors. These students have all been taught in schools implementing the EM curriculum across the board. The test administered is not the entirety of an existing standardized

test administered in the regular fashion under a real whole test situation. Rather, the test is prepared by the authors themselves by pulling problems selectively from a test used by another group of researchers back in 1995 (referenced by the group as two presentations made by Okamoto, Y., et. al. (1995, April), Children's intuitive understanding of number and formal mathematics learning: A cross-national comparison, and A developmental study of U.S. and Japanese children's representations of number, place value understanding, and mathematics achievement, at the annual meeting of the American Education Research Association, San Francisco). Fuson's group did not say which test it was, but claims it is like the National Assessment of Educational Progress (NAEP) test. A total of 24 problems were pulled from the test, and the results on these problems are compared between the EM students and students in the control group. Professor John Ray, a full professor with expertise in Evaluation, Assessment, and Testing Development in the Department of Education at The University of Tennessee says that pulling problems selectively from existing tests to make up a new test of one's own to test students is considered 'unprofessional'. The criticism here is whether the made-up 24-problems test is representative of a nationally normed standardized achievement test covering all areas of math competency, especially the computation skills, and whether they are ones that the authors predict the EM students would do well in.

What population of 2<sup>nd</sup> graders is used by the Fuson's group for the control group for the first study? This is where the second major flaw comes in. The authors have chosen a group composed of only 29 students from one single school (See table 1 and 2 of the first longitudinal study report ) whose characteristics are completely unreported by the authors (except that they are from the San Francisco area) to represent the 'U.S. traditional students'. Surely, a sample size of 29 from one single school in one geographic area is too small and localized to represent adequately the US traditional students. In addition, how can a comparison between 343 kids from 11 selected schools against a group of only 29 students of unknown background on an achievement test made up by the evaluators themselves be considered valid? A choice of sample size of 343 of known background against one of only 29 of unknown background is grossly unfair.

Now, let's get onto the second study reported by Fuson et. al in the same report, involving comparison of 236 3<sup>rd</sup> graders of EM students against 1800 of US traditional students of unknown background. Similar to the first study for the 2<sup>nd</sup> graders, the same group of EM kids are involved in this study. The test is again developed by the evaluators themselves by pulling selectively 22 questions from an existing test, the 4<sup>th</sup> NAEP standardized test, and 9 additional problems from 'a third-grade cognitively based mathematics test (Wood, T., and Cobb, P. 1989, *The development of a cognitively-based elementary school mathematics test: Final report*, West Lafayette, In: Purdue University, School Mathematics and Science Center)'. In addition, 33 problems are selected from the Everyday Math curriculum to see how the EM kids performed on them. Who are the control group for the NAEP and Wood & Cobb test comparisons? The authors relate that 'a total of 18,033 third graders participated in the fourth NAEP test. Only 10% to 15% of these students answered each question. A sample of 1,800 was assumed for the chi-square test (statistical

comparison of 2 results.)'. In other words, the authors do not know who these 1,800 students are, much less their background. Do they constitute a comparable sample adequately representing the mix of background represented by the EM kids? A sample of 1,800 students of unknown origin and background, surely cannot be regarded as to represent the 'US traditional students', as the Fuson group does in this study. As for the control group for the Wood-and-Cobb test comparison, a group of 191 students of unreported origin are used. Again, what kind of meaningful comparison of math achievement can be drawn on a test made up by the evaluators themselves by pulling problems from existing tests, and using control group of students of very unequal size of unknown characteristics? The results would be much more valid had the authors used a nationally recognized and normed standardized test and tracked the performance of the EM kids on this test for a number of years, and compare their scores to the national average over the same time period. In addition, comparison to the national average of the performance of the EM kids in other areas of achievement tests over the same time period should also be analyzed to ascertain that they are comparable to that of national average and therefore would not pose as being a superior group to begin with. Furthermore, scores for years past of students from the same 11 schools for second and third graders on the same standardized tests should also be used to serve as a baseline for comparing the present scores on the same test. However, this was not done and results from a contrived test was analyzed instead. At best, the results are to be regarded as useless.

In the second paper from the group concerning comparison of 170 4<sup>th</sup> graders of Everyday Math students is similarly flawed. The test is again made up by the authors by selectively pulling problems from NAEP test. The control group used is some 2000+ students who had taken the 4<sup>th</sup> grade NAEP tests elsewhere. But the profile and attributes of these 2000+ students are unreported by the group. The question that begs to be answered is again, are the 2000+ students in the control group similar in every way to the Everyday Math group of students except for the method of math instruction?

A final comment on the selection of the 343 2<sup>nd</sup> graders, 238 3<sup>rd</sup> graders and 170 EM 4<sup>th</sup> grader students used in the test group in each comparison is in order. The authors say that these are from 22 classes in 11 schools (for the 2<sup>nd</sup> and 3<sup>rd</sup> graders) and from 8 schools (the 4<sup>th</sup> graders). This would put an average of 16 and 21 students/class/school respectively. From the introduction section of the Fuson paper, the authors seem to suggest that the entire school district from which the test students are drawn has implemented the Everyday Math curriculum. Therefore, the selection of the 31 students (on the average) from each of 11 schools (and the 21 students from each of the 8 schools) can be such that, these 31(21) students come from the classes with the best teachers only. The authors do not elaborate on the selection process from each school (random or not). Dr. William Sanders, of the famed statistician who devised the Tennessee's value added assessment evaluation scheme has repeatedly reported that the effect of two to three years of teaching from good teachers on otherwise comparable students can make as much as 15-30% difference on normed standardized tests. This is from his years of observing and analyzing the TCAP scores of Tennessee school

children. Therefore, I submit that since the selection process is unreported, the 343 students (or the 238 students or the 170 students) could have been chosen from only the good teachers at each of the 11 (or 8) schools, therefore favoring the EM students in the comparison, especially in light of the completely unknown control group used in the two studies.

## **Summary and Conclusion**

In this article, I reviewed some of the salient features of the Everyday Mathematics curriculum, and the preferred algorithms favored by the curriculum in carrying out the four standard arithmetic operations. In addition, I listed some of the important skills not covered by the curriculum. Finally, I cited the flaws in 2 major longitudinal studies favoring the Everyday Mathematics curriculum, comparing the performance on math achievement tests of Everyday Mathematics kids against that of the 'U.S. tradition' kids. Given the lack of teaching of the traditional standard algorithms in the Everyday Mathematics curriculum, and the over emphasis of learning only the concepts and the lax attitude in requiring the mastery of skills, Everyday Mathematics should not be adopted for the Oak Ridge School System. Even with supplement, the curriculum should still not be adopted, because its cavalier attitude toward the teaching of the traditional skills, and the urging of kids in using any algorithm they discover and feel comfortable with. It would be much better instead, to adopt a solid mathematics text, and generously supplementing the curriculum with the use of manipulatives and experimentation. The teachers should definitely not start out by just teaching the skills required to solve problems, but rather to excite the students by showing the concept behind the operations being taught. Of course, this would involve continued teacher training to show and educate them the basic concepts behind the mathematical operations and algorithms, and perhaps workshops showing teachers when and where to bringing in the manipulatives to illustrate a concept. If the Everyday Math curriculum is adopted by the Oak Ridge schools and supplementing is left to the teachers themselves, huge variations in the completeness of supplementing to fill in the gaps may result. Professor Wayne Bishop, Professor of Mathematics at the California State University, Los Angeles, in reviewing the Everyday Mathematics curriculum in meeting the revised, much more rigorous new California Mathematics Standards concludes that (<http://mathematicallycorrect.com/bishop4.htm>)

*'In normal classrooms with normal teachers, I would characterize these materials (the second grade material) as "dangerous". My impression is that it would be very difficult to be sure that appropriate material has been covered adequately. One can expect a very high degree of teacher variability. Knowledgeable teachers, well grounded in the materials, may be able to pull it off;... There is almost no routine practice, although a small amount is built into the activities.'*

He further declares that

*'The fourth grade materials is even more dangerous than the second grade. I simply cannot tell if the Core Knowledge Sequence (those concepts and skills required by the new California State Standards) are being mastered or not. Everything is up to teacher competence to fill in the gaps, provide the routine drill, and confirm that it is happening. ...'*

In summary, it is recognized that we do have a problem at hand with the poor mathematics achievement level of some of the students in the Oak Ridge System (although it as a whole is already a lot higher than the state or national average), as there is across the board in the U.S., but changing over to the Everyday Mathematics curriculum, in my opinion, is not for the better. Let's consider this matter very carefully, taking in all the experiences of other communities and learn from their experiences and not reinventing our own wheels, and risk the deterioration of our superior educational achievements, which is an important contributing factor to the high quality of life we enjoy in Oak Ridge.

A final note: A good website address with articles on how to improve mathematical teaching at secondary schools, as well as teacher training programs is at

<http://mathematicallycorrect.com/teachers.htm>

<http://www.edexcellence.net/library/teacher.html>

## Appendix:

Excerpts from the Everyday Mathematics Teachers Reference Manual on alternative methods of performing the four mathematical operations of addition, subtraction, multiplication, and division, and documenting other important skills not taught

*(Everyday Mathematics Teacher's Reference Manual, Grades 4-6, The University of Chicago School Mathematics Project, Everyday Learning Corporation, Chicago, IL, 1999 ISBN 1-57039-515-2, pp. 127-139.)*

Figure 1. Alternative algorithms for addition and subtraction.

**Themes**

### Addition and Subtraction Algorithms

Of the dozens of possible addition and subtraction algorithms, the following are a few that are based on students' mental arithmetic efforts and search for procedures. These are merely examples that you may suggest to get some students started—you don't need to present all of them to your whole class.

**Addition Algorithms** The following algorithms work equally well for whole numbers and decimals, although the latter may require some extra careful handling if the addends don't have the same number of decimal places.

**Left-to-Right Algorithm**

- Starting at the left, add column-by-column; then adjust the result.
- Variation:* For some students the above process becomes so automatic that they start at the left and write the answer column-by-column, adjusting as they go without writing any in-between steps. If asked to explain, they say something like this: "200 plus 400 is 600, but (looking at the next column) I need to adjust that, so I write 7. 60 and 80 is 140, but that needs adjusting, so I write 5. 8 and 3 is 11, no more to do, just write 1."

**Partial-Sums Algorithm**  
Add the numbers in each column. Then add the partial sums.

**Opposite-Change Rule**  
If a number is added to one of the addends and the same number is subtracted from the other addend, the result remains the same. The aim is to rename the addends so that one of the addends ends in zeros.

- Rename the first addend, and then the second. For example:  

$$\begin{array}{r} 268 \rightarrow 270 \rightarrow 300 \\ + 483 \rightarrow + 481 \rightarrow + 451 \\ \hline \text{Add. } 751 \end{array}$$

Explanation: Adjust by 2, and then by 30.
- Rename the second addend, and then the first. For example:  

$$\begin{array}{r} 268 \rightarrow 261 \rightarrow 251 \\ + 483 \rightarrow + 490 \rightarrow + 500 \\ \hline \text{Add. } 751 \end{array}$$

Explanation: Adjust by 7, and then by 10.

For more, see "A Special Note on Adding and Subtracting Decimals," p. 130.

	2 6 8
	+4 8 3
1. Add.	6 14 11
2. Adjust 10s and 100s.	7 4 11
3. Adjust 1s and 10s.	7 5 1

Left-to-right algorithm

	2 6 8
	+4 8 3
	6 4 1
	7 5 1

A left-to-right variation

	268
	+ 483
1. Add 100s.	600
2. Add 10s.	140
3. Add 1s.	+ 11
4. Add partial sums.	751

Partial-sums algorithm

Figure 2. Finding a least common denominator is meaningless according to *Everyday Mathematics*.

***Fraction-Addition Algorithm***

In *Fifth Grade Everyday Mathematics*, students learn to find a “quick common denominator” for adding fractions by simply multiplying the denominators of the addends. For example:

$$\frac{1}{3} + \frac{5}{8} = \frac{1 * 8}{3 * 8} + \frac{5 * 3}{8 * 3} = \frac{8}{24} + \frac{15}{24} = \frac{23}{24}$$

This algorithm may be generalized for sixth graders as a formula, although students should not necessarily be expected to memorize or prove it.

$$\frac{a}{b} + \frac{c}{d} = \frac{(a * d) + (c * b)}{b * d}$$

It is important to note that *Everyday Mathematics* does not support the traditional emphasis on finding a *least* common denominator. This approach is excessive, too formal, and without much meaning for many people. In fact, implying the least common denominators are the only permissible denominators is probably harmful to later learning in algebra.

**Figure 3. Subtraction algorithms -- everything but the standard algorithm!**

See "A Special Note on Adding and Subtracting Decimals," p. 130.

**Subtraction Algorithms** As with addition algorithms, the following work for whole numbers and decimals.

**Add-Up Algorithm**  
 Add up from the subtrahend (bottom number) to the minuend (top number). For example:

$$\begin{array}{r} 932 \\ -356 \\ \hline \end{array} \quad \begin{array}{r} 356 \\ 360 \\ 400 \\ 900 \\ 932 \\ \hline \end{array} \begin{array}{l} + 4 \\ + 40 \\ + 500 \\ + 32 \\ \hline \end{array}$$

Add. 576

**Left-to-Right Algorithm**  
 Starting at the left, subtract column-by-column. For example:

$$\begin{array}{r} 932 \\ -356 \\ \hline \end{array} \quad \begin{array}{l} 1. \text{ Subtract the 100s.} \\ 2. \text{ Subtract the 10s.} \\ 3. \text{ Subtract the 1s.} \end{array} \quad \begin{array}{r} 932 \\ -300 \\ \hline 632 \\ -50 \\ \hline 582 \\ -6 \\ \hline 576 \end{array}$$

**Same-Change Rule**  
 If the same number is added to or subtracted from both the minuend and the subtrahend, the result remains the same. The aim is to rename both the minuend and the subtrahend so that the subtrahend ends in zero.

1. Add the same number. For example:
 
$$\begin{array}{r} 932 \rightarrow 936 \rightarrow 976 \\ -356 \rightarrow -360 \rightarrow -400 \\ \hline \text{Subtract. } 576 \end{array}$$

Explanation: Adjust by 4, and then by 40.
2. Subtract the same number. For example:
 
$$\begin{array}{r} 932 \rightarrow 926 \rightarrow 876 \\ -356 \rightarrow -350 \rightarrow -300 \\ \hline \text{Subtract. } 576 \end{array}$$

Explanation: Adjust by 6, and then by 50.

**Figure 4. More subtraction algorithms.**

Subtract. 576

$$\begin{array}{r} 932 \\ -356 \\ \hline \end{array}$$

1. Subtract 100s: 900 - 300 = 600
2. Subtract 10s: 30 - 50 = -20
3. Subtract 1s: 2 - 6 = -4
4. Add the partial differences. 576 (600 - 20 - 4, done mentally)

Explanation: Adjust by 6, and then by 50.

**Partial-Differences Algorithm**  
 Write partial differences, negative if necessary, and adjust. A few students who love negative numbers use some variation of the procedure shown in the margin.

**An Unusual Subtraction Algorithm**  
 Subtract by adding from the subtrahend to the minuend, column-by-

Figure 5. Yet another unusual subtraction algorithm.

**An Unusual Subtraction Algorithm**

Subtract by adding from the subtrahend to the minuend, column-by-column, with adjustments. Some students who use the add-up algorithm use this variation. They just write the answer with no other remarks. Asked to explain, they say something like this:

“To get to 900 from 300, add 600; but the tens will need help, so write 5 [for 500]. Since  $500 + 300 = 800$ , this leaves 132 still to subtract from. To get to 130 from 50, add 80; but the ones will need help, so write 7 [for 70]. Since  $70 + 50 = 120$ , this leaves 12 still to subtract from. To get to 12 from 6, add (and write) 6. No more to do.”

Figure 6. *Everyday Mathematics* wishes decimal numbers would only come in an equal number of digits.

For more, see “Estimates in Calculations,” p. 149.

$25 + 1.726 + 0.05 = ?$

25.000	
1.726	
+ 0.050	
26.776	

“Evening out” the decimals

**A Special Note on Adding and Subtracting Decimals** In everyday life, most decimals to be added or subtracted tend to have the same *precision*, or number of decimal places. A common example is money, where most values are rounded to the nearest cent and computation is with values in the hundredths place or above. In fact, there are very few situations where you might need to combine or subtract values with different precisions, and even if a data set had such disparate measures, you would probably want to round them all to the same precision to minimize error.

In spite of this, students are likely to face a practical need to add or subtract disparate decimals in local, state, or national achievement tests. For this reason, students in fifth and sixth grade *Everyday Mathematics* are shown how to rename addends so that they all have the same number of decimal places—by inserting zeros to “even out” the decimals. Writing the values vertically while aligning the decimal point can make this simpler as is shown in the example in the margin.

While students learn to cope with artificial testing demands in this way, the authors of *Everyday Mathematics* believe that they should do so in the full knowledge that these devices *are* artificial, and that in their own work with measures, they should apply the more sensible and realistic rules that all data in a problem should be collected and/or rounded to about the same level of precision, and that calculators can help improve accuracy.

Figure 7. Introduction to multiplication algorithms -- invent your own.

### **Multiplication Algorithms**

Sensible adults reach for calculators whenever they need to multiply with numbers that are not “easy.” The calculator option should also be available to students when they deal with problems that they may very well understand, but that involve calculations beyond their current skills. Yet, as always, *Everyday Mathematics* suggests that you first encourage students to share the ways in which they find products of whole numbers by mental arithmetic, and to invent and use their own systematic computational procedures.

It may be more difficult to invent multiplication procedures than addition and subtraction procedures, but when estimating and doing mental arithmetic, many students begin to add partial products: “Ten of these would be... , so 30 of them would be ... , and then we need 5 more, so... .” Beginning in *Third Grade Everyday Mathematics*, the partial-products algorithm is suggested as an initial approach to finding products using formal paper-and-pencil procedures.



**Figure 9. Multiplication by partial products – continued.**

With the rule  
 $(a + b)^2 = a^2 + 2ab + b^2$ .  
 $25^2$  can be calculated as  
 $(20 + 5)^2 = 400 + (2 * 100) + 25$   
 $= 625$ .

With the rule  
 $(a + b)(a - b) = a^2 - b^2$ .  
 $23 * 17$  can be calculated as  
 $(20 + 3)(20 - 3) = 400 - 9 = 391$ .

products added. For example:

$$(x + 2) * (x + 3) = (x * x) + (2 * x) + (3 * x) + (2 * 3)$$

$$= x^2 + 5x + 6$$

There are many multiplication algorithms other than the partial-products algorithm and the traditional right-to-left one. Former University of Chicago student Dan Hirschhorn, after only a few hours' search of old school books and mathematics education articles, found more than 40 different multiplication algorithms. About 25 of them were either special "tricks" for quick mental multiplication of numbers with special characteristics or procedures you may remember from high school algebra. Some are very efficient, but difficult to explain. More than 15 of the 40 were general algorithms for any two pairs of whole numbers, including those that follow, which are rather easy to illustrate.

67

53

**Figure 10. The lattice method for multiplication – part 1.**

The lattice method

	6	7	
3	3	0	3
5	1	8	1
5			1

$67 * 53 = 3551$

**The Lattice Method**

In 1478, the lattice method appeared in Treviso, Italy, in what is said to be the first printed arithmetic book. It was in use long before that, with some historians tracing it to Hindu origins in India before 1100 A.D. We include this algorithm mainly for its recreational value, historical interest, and the fact that it provides fine practice with multiplication facts and adding single-digit numbers. It is a very efficient algorithm, no matter how many digits are in the factors. (We have found that many students in *Everyday Mathematics* prefer this algorithm over all others.)

For example, to multiply 53 by 67:

- Draw a rectangular grid with as many rows as the number of digits in one factor and as many columns as the number of digits in the other factor. (2 by 2 in the example)
- Sketch a diagonal in each cell of the grid, from bottom left to top right.

Figure 11. Part 2 of the lattice method.

3 5 3

1	3	5	3	4	
1	1	2	2	0	2
6	2	1	3	5	2
7	1	5	2	5	1
8	1	8	3	0	1
	8	6	8		

353 \* 4756 = 1,678,868

For more see ►  
"The Distributive Property," p. 8 and  
"Multiplication Wrestling," p. 289

- Write the factors on the outside of the grid, one digit per cell. Left to right for the horizontal factor, top to bottom for the vertical factor.
- Multiply each digit in one factor by each digit in the other factor and enter the products in the two triangles of the cell corresponding to the pair of digits. If the product is less than 10, enter 0 in the top triangle and the product in the bottom one.
- Beginning in the bottom right triangle of the grid, add the numbers inside the lattice along each diagonal strip and write the result at the open end of the strip (along the bottom and left side of the grid). If the sum in a strip exceeds 9, add the excess 10s to the next diagonal strip up (as if you were carrying a number in the traditional right-to-left addition algorithm).
- Read the digits in the answer in order down the left side and across the bottom.

To explain why the lattice method works, the following example applies the distributive property as it is used in the *Multiplication Wrestling* game and in the Partial-Products Algorithm described earlier in this section.

To multiply:  $53 * 67$

First, rewrite the factors:  $(50 + 3) * (60 + 7)$

Next, expand, using the distributive property:  $(50 + 3) * 60 + (50 + 3) * 7$

Then use the distributive property twice more:  $(50 * 60) + (3 * 60) + (50 * 7) + (3 * 7)$

Multiply:  $3000 + 180 + 350 + 21$

Add:  $3551$

The addition is shown in vertical form in the margin.

Now look at the numbers in each diagonal strip in the lattice. The bottom right triangle contains the number of 1s in the sum of these four numbers. The numbers in the next strip up (8, 2, and 5) are the digits in the 10s place of the sum. The numbers in the next strip above that (1, 0, and 3) are the digits in the 100s place of the sum. Finally, the 3 in the upper left triangle is the number of 1000s in the sum. When you add along the diagonals, you are adding just as you would in the vertical form above.

In short, the lattice keeps track of the number of 1s, 10s, 100s, in some partial products without writing extra zeros. In this way, it shares one attribute of a traditional multiplication algorithm: the 0 in 3180 (the second partial product) is usually left out (see margin).

3000
180
350
+ 21
3551

53
* 67
371
318

Figure 12. The 4000 year old Egyptian algorithm for multiplication.

***An Egyptian Algorithm***

An algorithm for multiplication developed by the Egyptians over 4000 years ago eliminates the need for all multiplication facts except for the "2s." The idea of doubling, which students find easy and fun, is used repeatedly. This algorithm was used well into the Middle Ages, and a variation, called the Russian Peasant Method, is still used today in Russia, Ethiopia, the Arab world, and the Near East.

**Figure 13. The Egyptian algorithm continued – requiring the binary representation of one of the numbers being multiplied.**

Here is how to use the Egyptian method for  $28 * 13$ :

**Step 1:** List the consecutive powers of 2 beginning with 1. Stop with the power of 2 that is less than or equal to the smaller factor.

**Step 2:** In the second column, write the other factor next to the 1, and then double the factor repeatedly, stopping with the last power of 2 in the first column. Each number in the second column is the product of the number in the first column multiplied by the second factor.

For example,  $112 = 4 * 28$ .

**Step 3:** Check off the powers of 2 whose sum is the smaller factor.  
( $1 + 4 + 8 = 13$ )

**Step 4:** Cross out the rows that are not checked off.

**Step 5:** Add the numbers in the second column that are not crossed off.  
( $28 + 112 + 224 = 364$ )

Modern notation		Egyptian notation	
1st Column	2nd Column	1st Column	2nd Column
✓ 1	28	✓	
<del>2</del>	<del>56</del>		<del></del>
✓ 4	112	✓	
✓ 8	224	✓	
13	364		

The application of the distributive property shows why this method works.

$$13 * 28 = (1 + 4 + 8) * 28 = (1 * 28) + (4 * 28) + (8 * 28)$$

Source: Nelson, David, George Gheverghese Joseph, and Julian Williams. *Multicultural Mathematics*. Great Britain: Oxford University Press, 1993.

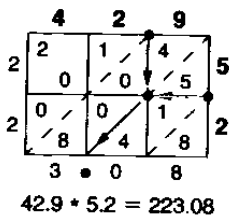
Figure 14. Roundabout methods for decimal multiplication – or use a calculator.

**Multiplying Decimals** Students of *Sixth Grade Everyday Mathematics* are encouraged to find their own algorithms for multiplying decimals in the same way they have been asked to find algorithms for operations with whole numbers and fractions. The primary approach suggested in the program is to estimate a product and then check for proper position of the decimal point after applying some other multiplication algorithm. There are a few ways to do this:

- First, estimate the size of the product. For example,  $0.23 * 17.56$  is about twice as much as  $0.1 * 17.56$ , so the product is about 4; or using fractions,  $0.23 * 17.56$  is about  $\frac{1}{4}$  of 16 or  $\frac{1}{5}$  of 20, and either of these estimates suggest a product of about 4.

Then use whatever whole-number multiplication method you prefer, ignoring the decimal points in the factors. Finally, use your initial estimate to figure out where to put the decimal point.

- Use a calculator. Verify the size of the answer in the display, perhaps using one of the approaches in the previous method, because even experienced calculator users occasionally make mistakes. Also check for accuracy, because the calculator may round the product to fit its digits in the display.
- Multiply as if the factors were whole numbers, and use a rule for placing the decimal point in the answer. For example, count the numbers of decimal places in the factors, add them, and mark off that many decimal places from the ones digit in the whole-number answer.



All of these methods require knowing some whole-number algorithm.

The lattice method can be used to multiply decimals. Simply find the intersection of the decimal points along the horizontal and vertical lines; then slide it down its diagonal.

Figure 15. Everyday Mathematics believes teaching long division is a waste of time.

### **Division Algorithms**

The authors of *Everyday Mathematics* do not believe it is worth the time and effort to fully develop paper-and-pencil long division algorithms for all possible whole number, fraction, and decimal problems. Mastery of the intricacies of these algorithms is a huge undertaking, and one that experience tells us is doomed to failure for many students. It simply does not seem wise to invest 100 hours or more on instruction and practice of algorithms that some students will succeed at doing slowly on paper, with uncertain results, while nearly anyone can quickly and accurately find a quotient with a calculator.

This said, the practical needs of students to succeed on outdated standardized tests may require you to teach long division algorithms. Toward this end Grades 4–6 of *Everyday Mathematics* contain several lessons aimed at helping students develop some paper-and-pencil division algorithms.

Figure 16. The "Guess-and-Check" approach to division!

For more on equal-sharing problems, ►  
see "Rate Diagrams," p. 98.

$$\begin{array}{r}
 12 \overline{)158} \qquad 10 \\
 \underline{120} \\
 38 \qquad 3 \\
 \underline{36} \\
 2 \qquad 13
 \end{array}$$

158:12 → 13R2

When we express the result of division in terms of a quotient and remainder, we use an arrow rather than an equals sign because 158:12 = 13R2 would not be a proper number sentence.

**A Guess-and-Check Division Algorithm**

Equal-sharing division problems ask, "How many of *these* are in *that*?" or, more symbolically  $a/b$  means "How many  $bs$  are in  $a$ ?" One approach to solving such problems is to use an algorithm that begins with a series of "at least, but less than" estimates of how many  $bs$  are in  $a$ . Next, check each estimate—if you have not taken enough  $bs$  from the  $a$ , then take some more. When you have taken all the  $bs$  there are, add the interim estimates.

For example, "How many groups of 12 are in 158?" can be solved by finding 158/12 groups. You might begin with multiples of 10, because they are simple to work with: There are at least ten 12s in 158 (that is,  $10 * 12 = 120$ ), but there are fewer than twenty ( $20 * 12 = 240$ ).

Record 10 as your first estimate and subtract ten 12s from 158, leaving 38. The next question is, "How many 12s are in the remaining 38?" You might know the answer right away ( $12 * 3 = 36$ ), or you might sneak up on it: "More than 1, more than 2, a little more than 3, but not as many as 4...." Taking out three 12s leaves 2, which is less than 12, so you can stop estimating. The final result is the sum of your estimates (13) plus whatever is left over (the remainder, 2). Of course, you may make a different series of estimates than were made above.

This guess-and-check division algorithm is introduced in *Fourth Grade Everyday Mathematics* and reviewed in Grades 5 and 6. Division with 2-digit divisors, which is introduced in Grade 5 and practiced in Grade 6, can be done by adapting the algorithm as necessary.

**Figure 17. Everyday Mathematics claims the 70+% of ORHS graduates planning to go to college do not need to learn the skill of dividing by fractions. They are badly misinformed. Have they taken a science, engineering, or mathematics course in college lately?**

**Dividing Fractions** Situations that involve quotients of rational numbers in *decimal* form are fairly common in everyday life. However, the same kinds of situations involving quotients of rational numbers in *fraction* form are rare. Indeed, few adults ever need to divide fractions once they leave school. Therefore, division of fractions in *Everyday Mathematics* is not provided to give students a practical skill, but to achieve the following:

- Complete the arithmetic system. Because whole numbers and decimals can be added, subtracted, multiplied, and divided, students should see that all these operations are also possible with fractions.
- Anticipate students' future work with rational number expressions in algebra courses.

**Figure 18. Simplification of fractions (finding greatest common divisor and least common multiple) is a waste of time according to Everyday Mathematics.**

For more, see **"Fraction-Addition Algorithm,"** above

Students use factor trees to find greatest common factors of two numbers, and, optionally, least common multiples and least common denominators. The latter two activities are optional because the authors of *Everyday Mathematics* believe students have more important, and certainly more interesting, things to do than waste hours writing fractions in an artificially "best" or "simplified" form.