Re: "Review of the Everyday Mathematics Curriculum and Its Missing Topics and Skills," by Tsewei Wang, PhD, Associate Professor of Chemical Engineering, University of Tennessee April 9, 2001

A. Isaacs, UCSMP, Director of the California edition of *Everyday Mathematics*

April 14, 2009

Professor Wang has several complaints about EM. First, she believes the research evidence of EM's effectiveness is faulty. Second, she opposes EM's approach to paper-and-pencil computation. Third, she claims that EM omits certain key topics that are important for success in higher mathematics.

Evidence of EM's Effectiveness

I will begin with Wang's claim that the research evidence for EM effectiveness is flawed. This, after all, bears on the key question: Does EM work?

The first thing to note is that Wang does not cite evidence of poor or declining test scores with EM, which one would think would be available if the program were as seriously flawed as Wang claims. The reason Wang fails to cite research evidence for EM's lack of efficacy is that such research is not available. The many research studies of EM all point in the same direction: Test scores tend to rise with EM. Since Wang cannot cite evidence of poor results with EM, she must instead attack the credibility of the many studies that show good results with EM.[1]

Second, note that even if Wang's critique of the studies of EM were valid, she would have shown only that the studies were flawed, not that EM is flawed. A study of a bad program can be a good study; a study of a good program can be a bad study. Criticizing studies of EM is not equivalent to criticizing EM.

Third, the question of the quality of the studies of EM has been considered at length both by a panel appointed by the National Research Council and by panels at the U.S. Department of Education's What Works Clearinghouse. The judgments of these panels, which were each made of up experts in the evaluation of educational programs, should be

[1] This is an example of what is known as confirmation bias. Confirmation bias occurs when someone has a predetermined conclusion and seeks out evidence to support that conclusion. One suffering from confirmation bias also tries to discount evidence that the desired conclusion is wrong. This, of course, is the opposite of the way science is supposed to work. One is supposed to reach one's conclusions after looking at the evidence, not seek out evidence for the conclusions one already has. Professor Wang appears to have fallen victim to this error. She has failed to find evidence that the conclusion she wants is true – that is, she has failed to find research studies of EM that show poor results – so she is reduced to trying to discredit evidence that undermines her desired conclusion.

given much more serious weight than the critique of someone whose professional training is largely irrelevant to the issues.

In 2002, the National Research Council convened a panel of experts to examine the state of evaluations of the effectiveness of school mathematics curricula. This panel, chaired by Jere Confrey of the University of Texas, examined all the extant research on the effectiveness of a large sample of curricula at all levels, including EM. The panel looked at the wide variety of studies that are reported as ‘research,’ including both what one might call ‘armchair analyses’ and also actual empirical studies. Of the latter, some studies had no control or comparison group, and were deemed less informative for that reason. The highest quality and most important class of studies identified by the panel was “comparative studies.” To qualify as a comparative study, a study had to have credible evidence about student outcomes and some sort of comparison or control group.

At the elementary level, the panel identified 23 comparative studies of the five programs it examined, one of which was EM. Of these studies, 17 were of EM. Repeat: of the 23 comparative studies the NRC panel identified, 17 were of EM. That is, the vast majority of all comparative studies of elementary school mathematics curricula were of EM. The NRC panel then identified 18 of these 23 studies as “minimally methodologically adequate” (MMA), which essentially means that they were not as good as the NRC wanted, but were at least of high enough quality to consider. The NRC didn’t reveal which 18 studies were MMA and which 5 studies weren’t MMA, but even if all 5 studies that were not MMA were of EM, it would still be the case that 2/3 of the MMA studies were of EM. And one might suspect some of the non-MMA studies were of other curricula so that it is likely that more than 2/3 of the MMA studies were of EM. In any case, EM by itself has at least twice as many MMA studies as the other four curricula combined.

This testifies to the overall quantity and quality of studies of EM and thus refutes the main point of Wang’s attack on the quality of studies of EM. Wang’s claim that studies of EM are of poor quality is simply not supported by an independent analysis of research about EM and other elementary curricula. There are far more high-quality studies of EM than of any other elementary school mathematics curriculum.

What is even more interesting, however, is what the NRC concluded on the basis of the 18 MMA studies (and therefore on the basis of a body of research that was at least 2/3 about EM). Here’s a quote from the report:

> At the elementary level, the findings of the review of evaluations of data on the effectiveness of NSF-supported curricula report consistent patterns of benefits to students. Across the studies, it appears that positive results are enhanced when accompanied by adequate professional development and the use of pedagogical methods consistent with those indicated by the curricula. The benefits are most consistently evidenced in the broadening topics of geometry, measurement, probability, and statistics, and in applied problem solving and reasoning. …

> The data from these evaluations at the elementary level generally present credible evidence of increased success in engaging minority students and students in poverty based on reported gains that are modestly higher for these students than for the comparative groups. What is less well documented in the studies is the extent to which the curricula counteract the tendencies to see gaps emerge and result in long-term persistence in performance by gender and minority group membership as they move up the grades. However, the evaluations do indicate that these curricula can help, and almost never do harm.
> (NRC, 2004, pp. 159-160)

This is the finding of an independent panel of the most august scientific body advising the U.S. Government, the National Academy of Science. The citation is

> National Research Council. (2004). On evaluating curricular effectiveness: Judging the quality of K-12 mathematics evaluations. Committee for a Review of the Evaluation Data on the Effectiveness of NSF-Supported and Commercially Generated Mathematics Curriculum Materials. Confrey, J. & Stohl, V. (Eds.), Mathematics Science Education Board, Center for Education, Division of Behavioral and Social Science and Education. Washington, DC: The National Academies Press.

More evidence that the quality of the research on EM is high can be found a series of reports from the U.S. Department of Education's What Works Clearinghouse. The WWC has issued five reports on elementary school mathematics curricula:[2]

> Everyday Mathematics
> Progress in Mathematics
> Scott Foresman-Addison Wesley Mathematics
> Saxon Elementary School Math
> Houghton Mifflin Math

The only program to be found to have evidence of positive results was EM. That is, EM is the only elementary mathematics program found by the U.S. Department of Education to have scientific evidence of positive effects.

This result also directly contradicts Wang's claim that evidence of EM's effectiveness is flawed. WWC's evidence screen – the criteria it applies to decide whether a study is trustworthy – is extremely rigorous. Most programs, in fact, do not have any studies that qualify.

One further point. Note that Professor Wang's critique is from 2001. In more recent attacks on EM, one does not see much discussion of the evidence for or against EM. That's because that argument is largely over – and the opponents of EM lost. Instead

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[2] These reports are available at
http://ies.ed.gov/ncee/wwc/reports/topic.aspx?tid=04

opponents of EM are reduced to anecdotes and armchair analyses that purport to show that the program is seriously flawed.[3]

Algorithms for Paper-and-Pencil Computation

The bulk of Wang's review is devoted to criticizing EM's approach to paper-and-pencil calculation. Her technique is to set up a straw man by distorting how EM actually approaches computation and then to knock down that straw man. She suggests, for example, that EM supports efforts to eliminate paper-and-pencil computation from the school mathematics curriculum (p. 5). That this claim is false should be clear to anyone who actually looks at the EM books, which are filled with instruction and practice in paper-and-pencil arithmetic. Wang also repeatedly claims that EM students are asked to invent their own methods for carrying out computations, another claim that is easily seen to be false by looking at the books, which have many worked examples of various approaches to carrying out calculations. We do, indeed, present several alternative algorithms for each operation, but we do not expect students to "discover" their own algorithms.[4]

We suggest alternative algorithms for various reasons. Some are particularly well suited to mental arithmetic and computational estimation, skills that are increasingly important in a world in which much routine calculation is done by machines. People need these skills to be able to judge whether the number on the cash register is accurate. Other algorithms are conceptually more transparent that the traditional approaches. The partial-products multiplication algorithm, for example, is more conceptually transparent than the traditional long multiplication algorithm and also provides much useful practice in "extended multiplication facts" that are essential in computational estimation.

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[3] In an earlier draft of this review of EM, Wang correctly stated that an important criterion for judging the quality of research is whether it has been published in a peer-reviewed journal. We agree. This is an important criterion – and one that research on EM meets: Many research studies of EM have been published in peer-reviewed journals. Wang has dropped that criterion here, probably because she has become aware of the large body of research about EM in the peer-reviewed literature. Note however, that not one of Wang's "sources" have been peer reviewed. On the other hand, many positive studies of EM have been published in peer-reviewed journals -- see

http://everydaymath.uchicago.edu/about/research/

for summaries and bibliographies of these studies. The peer-review process is essential for sorting out essentially baseless attacks on EM generated by armchair analyses of the materials by people who have no expertise in elementary mathematics education from serious research done by responsible people and published in respected journals. Anybody can write something and post it on the web; that doesn't make it research.

[4] For details about the EM approach , see the white paper on algorithms in EM at http://everydaymath.uchicago.edu/about/research/algorithms.pdf

Note that this approach is not unusual in mathematics. In computer science, for example, one routinely studies many algorithms for sorting. Some of these algorithms are not practical but are conceptually important. Others are practical in certain situations, with small numbers of items, for example, but not in other situations. In general, however, there is no single sorting algorithm that is universally optimal.

Wang seems to think that in elementary school mathematics, unlike in computer science or many other fields, there are universally optimal algorithms, which are more or less those that have been widely taught in the past 50 years or so in the United States. The EM authors, on the other hand, believe students should master a variety of methods so that they can choose the one most appropriate to the situation at hand.

Two further points. First, Wang's critique is based on the second edition of EM, not the California edition, which is based on the national third edition. So, for example, Wang complains that the U.S. traditional algorithms are not included in the program. But this is not true of either the third edition or the California edition. Second, note that Wang does not cite any evidence about how EM students perform on standardized tests of computation. This is because EM students do just fine on standardized tests of computation. They may not solve the problems the way Wang would or the way their grandparents would, but they do solve them and they solve them correctly.

Omitted Topics

Wang claims that EM omits certain topics that are essential to success in higher mathematics. She identifies three such topics, long division, multiplication of decimals, and division with fractions. Note that two of these three "key topics" are really just a repeat of the complaints Wang has about how EM approaches computation. So beyond her complaints about how EM teaches paper-and-pencil arithmetic, her list of omitted topics is quite short: division with fractions.

In any case, Wang is wrong to claim that EM omits these topics: EM does include long division, including the U.S. traditional division algorithm; EM does include long multiplication with decimals, again including the U.S. traditional multiplication algorithm; and EM does include extensive work with fraction division, including the traditional invert-and-multiply approach.

The last half of Wang's paper is an appendix that critiques excerpts from the EM materials, most of which have to do with computational algorithms. Sometimes she criticizes a position EM takes that differs from the standard approach even when the EM approach makes more sense. Other times she simply distorts the EM position to make a rhetorical point. In general, Wang's criticisms are unfounded. Consider just three.

Wang's caption for Figure 2 states, "Finding a least common denominator is meaningless according to Everyday Mathematics." The actual excerpt from EM in the figure shows that this is not what EM says. The EM Authors believe that much of the fraction work in schools is "too formal and without much meaning for many people." Is this not true? If you think it's not true, I suggest you stop a random adult on the street and ask him or her to perform a simple calculation with fractions such as 1¾ ÷ ½. Even if the person is able to do the calculation, which is doubtful, it is extremely unlikely that the person would be able to explain what it means – as Liping Ma has so ably shown in her recent book, *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. If Wang thinks the traditional approach to fraction computation is working, she's quite mistaken, as anyone who has taught elementary school mathematics can confirm. On the other hand, EM's Quick Common Denominator method, which Wang opposes, has significant advantages in ease of use, in conceptual transparency, and in preparing students to succeed in algebra.

Wang's caption for Figure 6 is "Everyday Mathematics wishes decimal numbers would only come in an equal number of digits." Again, this is a distortion of what EM actually says, which is that in everyday life most decimals one would perform calculations on are already provided with a fixed number of places – and also that EM teachers kids how to transform decimals that do not have equal numbers of places into those that do. Again, is this not the case? Look around for decimals in the world. How often do you find sets of decimals in which there are different numbers of places? Not very often, which is what EM says and Wang apparently doesn't believe.

Wang's caption for Figure 17 is "Everyday Mathematics claims the 70%+ of ORHS graduates planning to go to college do not need to learn the skill of dividing by fractions. Have they taken a science, engineering, or mathematics course in college lately?" First, the EM authors have, indeed, taken advanced classes – my doctorate is in mathematics, not mathematics education, and the director of the second edition has a doctorate in statistics. My guess is that the EM authors are more qualified in mathematics than any other group of elementary school mathematics textbook authors in the country. Second, the claim about EM in the caption is completely wrong, as can be seen from reading the excerpt in the figure, which simply says that fraction division should be taught not for its utility in everyday life – no one does much fraction division in everyday life, after all – but rather for its importance in algebra and higher mathematics. So EM does not deemphasize fraction division, as Wang claims.

Other Points

Wang makes a number of other assertions in her review that require some response.

- Wang's section on the "History of the Emergence of the Constructivist Mathematics Curriculum" contains a number of inaccuracies. First of all, there is no such thing as "the constructivist mathematics curriculum." Constructivism is an epistemology, a theory of what it means to know something. It is not even a theory of learning in the narrow sense. It is certainly not a theory of teaching. There are a number of curricula that hold that it's neither necessary nor advisable to tell students everything, but to lump all such curricula together under the rubric "constructivist mathematics curriculum" is misleading.

In the same section, Wang claims that the 1989 NCTM's *Curriculum and Evaluation Standards for School Mathematics* "are regarded by professional mathematicians as watered down and, among other things, inadequate in its not teaching the standard mathematical algorithms of addition, subtraction, multiplication, and division." This is false, as one can easily verify by looking at the front matter of the document in question. There you will find endorsements from dozens of professional societies, including every major society of professional mathematicians. Some mathematicians do, indeed, oppose the NCTM Standards, but some is far from all. The vast majority, I'm sure, have no opinion of the NCTM Standards. The few who do have an opinion are divided, which is not the message Wang is attempting to convey.

Finally, in the same section, Wang states that the "NSF formally endorsed 10 curriculums as 'exemplary' or 'promising,'" including EM. This is not accurate – it was the U.S. Department of Education that produced the exemplary and promising list (pursuant to a mandate from Congress).

All three of these errors appear in a single paragraph. One error is minor, but the other two are not, and the fact that Wang gets so much wrong in a single paragraph reveals how sloppy her "research" is. It's not enough to go to the Mathematically Correct web site and believe whatever is written there.

- Wang refers to David Klein's Open Letter to Secretary Riley about the Department of Education's endorsement of certain curricula as exemplary or promising. She goes on at some length about the many famous mathematicians and scientists who signed the letter. What she does not mention is that the letter was condemned by Hyman Bass, then the president of the most important association of professional mathematicians, the American Mathematical Association, and that Leon Lederman, a Nobel laureate in physics and a signer of the letter, has withdrawn his signature because, as he wrote in a letter to Professor Klein, "I totally misunderstood the issues, did not take the time to understand them even to the point of misunderstanding positions as between the NCTM standards and its critics." I suspect there are other such signers who now regret signing the letter but who lack Dr. Lederman's courage in admitting a mistake.[5]
- Wang mentions several districts where there has been opposition to EM. So far as we know, all of these districts continue to use EM, which suggests that perhaps things are not so bad. Wang's argument would be more persuasive if she were able to cite districts where results with EM were poor and the program was accordingly dropped. That the districts she singles out, such as Reading MA, still use the program is revealing.
- Wang criticizes EM for "an overemphasis of the development of the concepts of mathematics…" It is, of course, always bad to over-emphasize something, but given the overwhelming overemphasis of procedures and facts in school mathematics, it is likely that a proper balance between procedures, facts, and concepts would seem to some people like an overemphasis on concepts. What Wang really wants is a return to an intense focus on the memorization of facts

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[5] We can provide a fuller response to the Open Letter. Email us at em-center@lists.uchicago.edu

procedures, an approach that has been ineffective in getting many of our students to be able to think mathematically and solve problems.

Conclusion

Wang no doubt believes passionately that EM is a bad program. She has also expended considerable effort in collating so much “research” about the program, even though almost all of it is from web sites such as mathematicallycorrect.com whose credibility and impartiality may be doubted. But belief is not evidence and strenuous objections are not compelling arguments.

The evidence is that EM produces good results. That may be hard for Professor Wang and some others to believe. That EM works may not make sense to them because the EM approach is not the one they experienced when they were in school. But if you look at the evidence, if you look at how the program works in real schools and how real children achieve with it, you will find little support for Wang’s claims.